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Changes in the statistical properties of electromagnetic double multi-Gaussian Schell-model beams on propagation in free space

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Abstract: A class of electromagnetic random sources with multi-Gaussian functional form in the spectral density and in the correlations part of the cross-spectral density matrix is introduced based on the genuine cross-spectral density matrix theory. The analytic propagation formulas of the cross-spectral density matrix of such beams propagating in free-space are derived by use of the Collins' diffraction integral. With the help of analytic formulas, the evolution of statistical characteristics, i.e. the spectral density, the spectral degree of polarization and the spectral degree of coherence for such beams in free-space are analysed numerically. Employing the multi-Gaussian functional form in the cross-spectral density matrix introduces one more freedom in the modeling of Gaussian Schell-model sources.

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1. Introduction

Due to their significance in real applications, researchers have recently focused on partially coherent and partially polarized light beams propagating in free space and in turbulent atmosphere [1–3]. Wolf's unified theory of coherence and polarization [4] made it possible to identify the characteristics of light beams, particularly their spectral density, spectral degree of coherence (DOC), spectral degree of polarization (DOP), and polarization states.

However, the interaction with medium is inherently connected to all of the practical uses of different partially coherent light beams, including optical communications, imaging, sensing, and lidar. The expansion and wander of the beam, the loss of spatial coherence, and random fluctuations in the irradiance and phase are caused by the random variations in the medium's refractive index [5-6]. The propagation parameters of a light beam will alter in a random medium, which will also lower the beam quality. These impacts will require modifications on how light fields are used in practice. The modeling of random light beams to overcome these types of problems have thus been the subject of extensive research in recent years [see Ref. [7] for a review].

Partially coherent beams with nonconventional correlation functions [7], or with nonuniform correlation functions [8], that are connected with unique coherence functions are quite engaging since their propagation characteristics can be controlled by the coherence functions. First modification to Gaussian Schell-model random beams is performed in [9] based on the genuine cross spectral density function theory [10]. Then, the same theory is used to further investigate partially coherent light beams having different far-field spectral density distributions [11,12]. Our purpose, based on the genuine cross spectral density matrix, is to model electromagnetic double multi-Gaussian Schell-model beam (DMGSM) and to study its propagation in free space. We obtain all four cross-spectral density matrix components of the beam to analyse the spectral density, the spectral degree of polarization, and the spectral degree of coherence in free space propagation.

2. Electromagnetic double multi-Gaussian Schell model sources

Multi-Gaussian functional form is first introduced in [13], and in [14] for the intensity part of the cross-spectral density function, later it is extended to partially coherent case in [15]. Also, Korotkova introduced random sources for rectangular far fields by using two one-dimensional multi-Gaussian distributions in the source degree of coherence [16]. We will develop a random source in which both the intensity and the correlations part of the cross-spectral density function have the multi-Gaussian functional forms.

Let us consider a random electromagnetic beam moving in the vicinity of the z-axis. At position P($\boldsymbol{r}$) (see Fig. (1)), and at frequency ω, let $\{\boldsymbol{E}(\boldsymbol{r},\omega)\} \equiv \{E_x(\boldsymbol{r},\omega),E_y(\boldsymbol{r},\omega)\}$ be a statistical ensemble of the fluctuating components of the transverse electric vector in mutually orthogonal x and y directions. The 2×2 electric cross-spectral density matrix may then be used to describe the beam's second-order correlation features

$$\overleftrightarrow{W}(\boldsymbol{r}_1,\boldsymbol{r}_2,\omega) = \left[W_{ij}(\boldsymbol{r}_1,\boldsymbol{r}_2,\omega)\right] = \left[\langle E_i^*(\boldsymbol{r}_1,\omega)E_j(\boldsymbol{r}_2,\omega)\rangle\right], i = x,y; j = x,y, \quad (1)$$

where the asterisk in $E_i^*(\boldsymbol{r},\omega)$ is the complex conjugate of the field and the angular brackets stand for the ensemble average.. The formula below defines the components of the cross-spectral density matrix of the field produced by a planar, secondary electromagnetic Schell-model source located in the plane $z = 0$ (also referred to the source plane):

$$W_{ij}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2;\omega) = \sqrt{S_i(\boldsymbol{\rho}_1;\omega)}\sqrt{S_j(\boldsymbol{\rho}_2;\omega)}\mu_{ij}(\boldsymbol{\rho}_2 - \boldsymbol{\rho}_1;\omega), \quad (2)$$

here, $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$ are the source plane locations in two dimensions, $S_{i,j}$ are the spectral densities of the x and y components of the electric field, and $\mu_{ij}(\boldsymbol{\rho}_2 - \boldsymbol{\rho}_1;\omega)$ are the coefficients that describe the correlation between the Cartesian components $E_i(\boldsymbol{\rho},\omega)$ and $E_j(\boldsymbol{\rho},\omega)$ of the electric field at the points $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$.

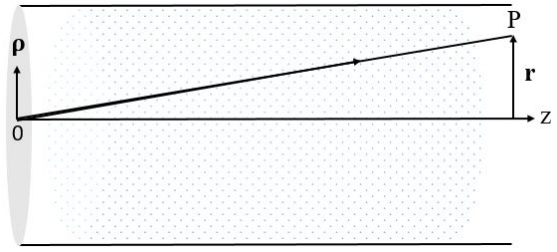


Fig. 1. Illustration of the notation.

Using correlations between the two spatial coordinates $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$, the cross-spectral density matrix $W_{ij}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)$ describes the spatial coherence characteristics of the field at the input plane. The crosss-pectral density function must match a nonnegative definite kernel for a correlation function to be considered genuine [10,17], and this condition is met if the function can be expressed in the form

$$W_{ij}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2) = \int p_{ij}(\boldsymbol{v})H_i^*(\boldsymbol{\rho}_1,\boldsymbol{v})H_j(\boldsymbol{\rho}_2,\boldsymbol{v})d^2\boldsymbol{v}, \quad (3)$$

where $p_{ij}(\boldsymbol{v})$ is a random nonnegative function which is Fourier transformable, and $H_{i,j}$ is a random kernel. For the sake of conciseness, Ref. [10,17] implied but did not explicitly state that the cross-spectral density depends on frequency ω. It has been demonstrated that Eq. (1) may be extended to electromagnetic vector fields and corresponds to a necessary and sufficient nonnegative definiteness requirement for any genuine cross-spectral density matrix [17]. It was shown in [17] that if the kernel $H_{i,j}$ is chosen as

$$H_i(\boldsymbol{\rho},\boldsymbol{v}) = A_i\tau(\boldsymbol{\rho})\exp[-i\boldsymbol{v}\cdot\boldsymbol{\rho}], \quad (4)$$

then the cross-spectral density, Eq. 2, takes on the following general form

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$$W_{ij}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2) = \tau_i^*(\boldsymbol{\rho}_1)\tau_j(\boldsymbol{\rho}_2)\tilde{p}_{ij}[\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2], \tag{5}$$

where $\tau(\boldsymbol{\rho})$ is a possibly complex profile function and $\tilde{p}_{ij}$ is the Fourier transform of the function $p_{ij}(\boldsymbol{v})$. As a particular case of the partially coherent fields based on this model, we consider multi-Gaussian functional forms for $p_{ij}(\boldsymbol{v})$ and for the kernel function

$$H_i(\boldsymbol{\rho},\boldsymbol{v}) = \frac{A_i}{C_{0L}}\sum_{l=1}^{L}(-1)^{l-1}\binom{L}{l}exp\Big[-\frac{l\boldsymbol{\rho}^2}{4\sigma^2}\Big]exp[-i\boldsymbol{v}\cdot\boldsymbol{\rho}], \tag{6}$$

$$p_{ij}(\boldsymbol{v}) = \frac{B_{ij}\delta_{ij}^2}{C_{0N}}\sum_{n=1}^{N}(-1)^{n-1}\binom{N}{n}exp\Big[-\frac{n\delta_{ij}^2|\boldsymbol{v}|^2}{2}\Big]. \tag{7}$$

In Eqs. (6) and (7) A_i is the amplitude, $B_{ij} = |B_{ij}|e^{\varphi_{ij}}$ will serve as the correlation coefficients of the x and y field components. Parameter σ is the transverse beam width of the source and δ_{ij}^2 are the variances across the correlations between the components of the electric field vector, respectively. And, the normalization constant C_0 depending on the summation is given by

$$C_{0L}=\sum_{l=1}^{L}\frac{(-1)^{l-1}}{l}\binom{L}{l},\ C_{0N}=\sum_{n=1}^{N}\frac{(-1)^{n-1}}{n}\binom{N}{n}. \tag{8}$$

On substituting Eq. (6) and the Fourier transform of Eq. (7) in Eq. (5) we obtain the electromagnetic double multi-Gaussian Schell-model source given by the following expression

$$W_{ij}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2) = \frac{A_iA_jB_{ij}}{C_{0L}C_{0M}C_{0N}}\sum_{l=1}^{L}(-1)^{l-1}\binom{L}{l}exp\Big[-\frac{l|\boldsymbol{\rho}_1^2|}{4\sigma^2}\Big]\times$$

$$\sum_{m=1}^{M}(-1)^{m-1}\binom{M}{m}exp\Big[-\frac{m|\boldsymbol{\rho}_2^2|}{4\sigma^2}\Big]\times\sum_{n=1}^{N}\frac{(-1)^{n-1}}{n}\binom{N}{n}exp\Big[-\frac{|\boldsymbol{\rho}_2-\boldsymbol{\rho}_1|^2}{2n\delta_{ij}^2}\Big]. \tag{9}$$

First, let us define the source parameter constraints that will ensure that the mathematical model (9) expresses a field that can be realized physically which are known as realizability conditions. It immediately follows that

$$B_{xx} = B_{yy} = 1, \qquad |B_{xy}| = |B_{yx}|, \qquad \delta_{xy} = \delta_{yx}, \tag{10}$$

from the requirement that the correlation matrix must be quasi-Hermitian [18]. Also, by employing Schwarz inequality, it is shown in [19] that for electromagnetic Schell-model sources (EM GSM), the modulus of B_{xy} is strictly less than one;

$$|B_{xy}| \le 1,\ x \ne y. \tag{11}$$

In obtaining electromagnetic double multi-Gaussian Schell-model source we take the isotropic variances, $\sigma_i = \sigma_j = \sigma$. Anisotropic variance situation deserves further consideration and is a place of another study.

The next step is to obtain realizability conditions for parameters δ_{xx}, δ_{yy}, δ_{xy}, and B_{xy} such that the required inequality [17]

$$p_{xx}(v)p_{yy}(v) - p_{xy}(v)p_{yx}(v) \ge 0, \tag{12}$$

holds. In [20] the same function $p_{ij}(\boldsymbol{v})$ is used in the same context and the realizability condition is already obtained for multi-Gaussian Schell-model sources, namely

$$\max\{\delta_{xx},\delta_{yy}\} \le \delta_{xy} \le \sqrt{\frac{\delta_{xx}\delta_{yy}}{|B_{xy}|}}, \tag{13}$$

which does not depend on the summation index N. Further, for classic EM GSM sources, it is shown in [19] that

$$\sqrt{\frac{\delta_{xx}^2 + \delta_{yy}^2}{2}} \le max\{\delta_{xx},\delta_{yy}\}. \tag{14}$$

Therefore, by employing Eq. (13) and Eq. (14), we find that the parameter B_{xy} has to fulfill the following realizability condition

$$|B_{xy}| \le \frac{2}{\frac{\delta_{xx}}{\delta_{yy}} + \frac{\delta_{yy}}{\delta_{xx}}}. \tag{15}$$

Lastly, we now follow chp. 5.6.4 of [18] to obtain the beam conditions on the parameters for double multi-Gaussian Schell-model source generates a higly directional beam. The necessary and sufficient condition for the source is given by the following inequality

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$$\frac{lm}{2(l+m)\sigma^2}+\frac{1}{n\delta^2}\ll\frac{2\pi^2}{\lambda^2},\qquad l,m,n=1,\ldots,L,M,N. \tag{16}$$

When using the source beam in the numerical computations caution must be on these conditions. These factors are crucial for the design of devices that produce highly directed beams for a variety of uses.

3. Propagation of the cross-spectral density matrix in free space

On free-space (paraxial) propagation, in addition to the Huygens-Fresnel integral, the relation between the cross-spectral density matrix in the source plane and that at points $(\boldsymbol{r_1},z)$ and $(\boldsymbol{r_2},z)$, is given by the Collins' diffraction integral [21,22]. Let us consider a double multi-Gaussian Schell-model beam propagating in free space from the plane $z=0$ into the half-space $z>0$. By using the paraxial and the electromagnetic form of the Collins' diffraction integral, the cross-spectral density matrix of the double multi-Gaussian Schell-model beams through free space can be expressed as

$$W_{ij}(\boldsymbol{r_1},\boldsymbol{r_2},z;\omega)=\left(\frac{k}{2\pi B}\right)^2$$
$$\iint W_{ij}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)exp\Big[-\frac{ik}{2B}\{A(\boldsymbol{\rho_1^2}-\boldsymbol{\rho_2^2})-2(\boldsymbol{r_1}\cdot\boldsymbol{\rho}_1-\boldsymbol{r_2}\cdot\boldsymbol{\rho}_2)+D(\boldsymbol{r_1^2}-\boldsymbol{r_2^2})\}\Big]d\boldsymbol{\rho}_1 d\boldsymbol{\rho}_2, \tag{17}$$

where $A(z)$, $B(z)$, $D(z)$ are the elements of the 2×2 ray transfer ABCD matrix, $k=2\pi/\lambda$ is the wavenumber, λ being the wavelength of light in vacuum, and $\boldsymbol{r}=(x,y)$ is the two-dimensional vector in the plane $z>0$. Calculations pertaining to the diffraction of the beam utilizing the relationship to geometrical paraxial properties of the system may be greatly simplified with the use of the ABCD matrix. Equation (17) demonstrates how the first-order optical system transforms the beam's cross-spectral density matrix from the source plane $z=0$ to the plane $z>0$. We use the double multi-Gaussian Schell-model beam as a model for the initial beam, and the cross-spectral density matrix's component elements have the form specified in Eq. (9). The elements of the ABCD matrix for free space propagation is given by

$$\begin{bmatrix}A & B\\ C & D\end{bmatrix}=\begin{bmatrix}1 & z\\ 0 & 1\end{bmatrix}. \tag{18}$$

On introducing the central abscissa coordinate system variables

$$\xi=\frac{\boldsymbol{\rho}_1+\boldsymbol{\rho}_2}{2};\ \xi'=\boldsymbol{\rho}_2-\boldsymbol{\rho}_1, \tag{19}$$

$$\boldsymbol{r}=\frac{\boldsymbol{r}_1+\boldsymbol{r}_2}{2};\ \boldsymbol{r}'=\boldsymbol{r}_2-\boldsymbol{r}_1, \tag{20}$$

and on subtituting from Eq. (9) for the source into the propagation integral Eq. (17), and performing the integral we obtain the elements of the cross-spectral density matrix in any plane $z>0$ to be given by the formula

$$W_{ij}(\boldsymbol{r},\boldsymbol{r}',z;\omega)=\frac{A_iA_jB_{ij}}{C_{0L}C_{0M}C_{0N}}\sum_{l=m=n=1}^{L,M,N}\frac{1}{\Delta_{ij}}(-1)^{l-1}\binom{L}{l}(-1)^{m-1}\binom{M}{m}\frac{(-1)^{n-1}}{n}\binom{N}{n}exp\left[-\frac{4\alpha\boldsymbol{r}^2}{\Delta_{ij}}\right]$$
$$exp\left[\left(\frac{ikD}{B}-\left(\beta+\frac{ikA}{B}\right)\frac{1}{\Delta_{ij}}\right)\boldsymbol{r}\cdot\boldsymbol{r}'\right]exp\left[-\frac{\gamma_{ij}\boldsymbol{r}'^2}{\Delta_{ij}}\right], \tag{21}$$

where

$$\alpha=\frac{1}{16}\left(\frac{l}{\sigma^2}+\frac{m}{\sigma^2}\right), \tag{22}$$

$$\beta=\frac{1}{4}\left(\frac{l}{\sigma^2}-\frac{m}{\sigma^2}\right), \tag{23}$$

$$\gamma_{ij}=\alpha+\frac{1}{2n\delta_{ij}^2}, \tag{24}$$

$$\Delta_{ij}=\left(\frac{B}{k}\right)^2\left[16\alpha\gamma_{ij}-\left(\beta+\frac{ikA}{B}\right)^2\right]. \tag{25}$$

Equation (21) with the parameters Eqs. (22-25) provide a generic equation for the 2×2 cross-spectral density matrix components of electromagnetic double multi-Gaussian Schell-

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model beam traveling through a paraxial ABCD system. We may interpret the parameter Δ_{ij} as an expansion coefficient. The method of integration is shown in the appendix of [23].

Following are the equations for the cross-spectral density matrix elements of electromagnetic double multi-Gaussian Schell-model beam travelling through free-space when the values of the ABCD matrix elements from Eq. (18) are substituted into Eq. (21) and Eqs. (22-25)

$$W_{ij}(\boldsymbol{r},\boldsymbol{r}',z;\omega) = \frac{A_i A_j B_{ij}}{C_{0L}C_{0M}C_{0N}} \sum_{l=m=n=1}^{L,M,N} \frac{1}{\Delta_{ij}} (-1)^{l-1}\binom{L}{l}(-1)^{m-1}\binom{M}{m}\frac{(-1)^{n-1}}{n}\binom{N}{n} exp\left[-\frac{4\alpha \boldsymbol{r}^2}{\Delta_{ij}}\right]$$

$$exp\left[\left(\frac{ik}{z} - \left(\beta + \frac{ik}{z}\right)\frac{1}{\Delta_{ij}}\right)\boldsymbol{r}\cdot\boldsymbol{r}'\right] exp\left[-\frac{\gamma_{ij}\boldsymbol{r}'^2}{\Delta_{ij}}\right], \quad (26)$$

where

$$\Delta_{ij} = \left(\frac{z}{k}\right)^2\left[16\alpha\gamma_{ij} - \left(\beta + \frac{ik}{z}\right)^2\right], \quad (27)$$

and the parameters α, β, and γ_{ij} are given by Eqs. (22-24).

4. Spectral density changes of electromagnetic double multi-Gaussian Schell-model beams on propagation through free space

Numerous works have studied how a scalar source's spectrum can vary due to propagation, with the changes depend on the source's coherence characteristics (see Ref. [24] for preliminaries). The goal of this current section is to investigate the spectrum characteristics of double multi-Gaussian Schell-model beams that is travelling through free space. Numerical examples illustrate how the spectral properties of the double multi-Gaussian Schell-model beams propagating through free space can vary.

$S(\boldsymbol{r},z;\omega)$ is the spectral density which is proportional to the averaged electric energy density of the field defined by

$$S(\boldsymbol{r},z;\omega) = \mathrm{Tr}\,\overleftrightarrow{W}(\boldsymbol{r},\boldsymbol{r},z;\omega) = W_{xx}(\boldsymbol{r},\boldsymbol{r},z;\omega) + W_{yy}(\boldsymbol{r},\boldsymbol{r},z;\omega). \quad (28)$$

As can be seen from Eq. (28), the cross-spectral density matrix's diagonal elements are the sole factors that affect the spectrum at a given position $\boldsymbol{r}$. The diagonal elements of the cross-spectral density matrix change when $\boldsymbol{r}_1 = \boldsymbol{r}_2 = \boldsymbol{r}$ is chosen in Eq. 26, and found to be

$$W_{ii}(\boldsymbol{r},\boldsymbol{r},z;\omega) = \frac{A_i^2}{C_{0L}C_{0M}C_{0N}} \sum_{l=m=n=1}^{L,M,N} \frac{1}{\Delta_{ii}} (-1)^{l-1}\binom{L}{l}(-1)^{m-1}\binom{M}{m}\frac{(-1)^{n-1}}{n}\binom{N}{n} exp\left[-\frac{4\alpha \boldsymbol{r}^2}{\Delta_{ii}}\right], \quad (29)$$

and the parameters also change specific to $\boldsymbol{r}_1 = \boldsymbol{r}_2 = \boldsymbol{r}$ in the diagonal case

$$\gamma_{ii} = \alpha + \frac{1}{2n\delta_{ii}^2}, \quad (30)$$

$$\Delta_{ii} = \left(\frac{z}{k}\right)^2\left[16\alpha\gamma_{ii} - \left(\beta + \frac{ik}{z}\right)^2\right], \quad (31)$$

where the parameters α, and β, are given by Eqs. (22-23).

Fig. 2 shows the propagation of the spectral density (Eq. 29 in Eq. 28) of the Gaussian Schell-model fields in free space. $L = 1, M = 1, N = 1$ represents the Gaussian Schell model beam, $L = 1, M = 1, N = 4$ represents the multi-Gaussian Schell-model beam and $L = 4, M = 4, N = 1$ with $L = 4, M = 4, N = 4$ represent the double multi-Gaussian Schell-model beam. Comparison of Fig. 2(a) and Fig. 2(b) shows a redistribution in the spectrum of Gaussian Schell-model beam on passage to multi Gaussian Schell-model beam, Fig. 2(c) shows an additional redistribution in the spectral density on passage from multi-Gaussian Schell-model beam to double multi-Gaussian Schell-model beam. This indicates that the double multi-Guassian Schell-model beam may mitigate the spectral loss of multi-Gaussian Schell-model beam on propagation in linear random media.

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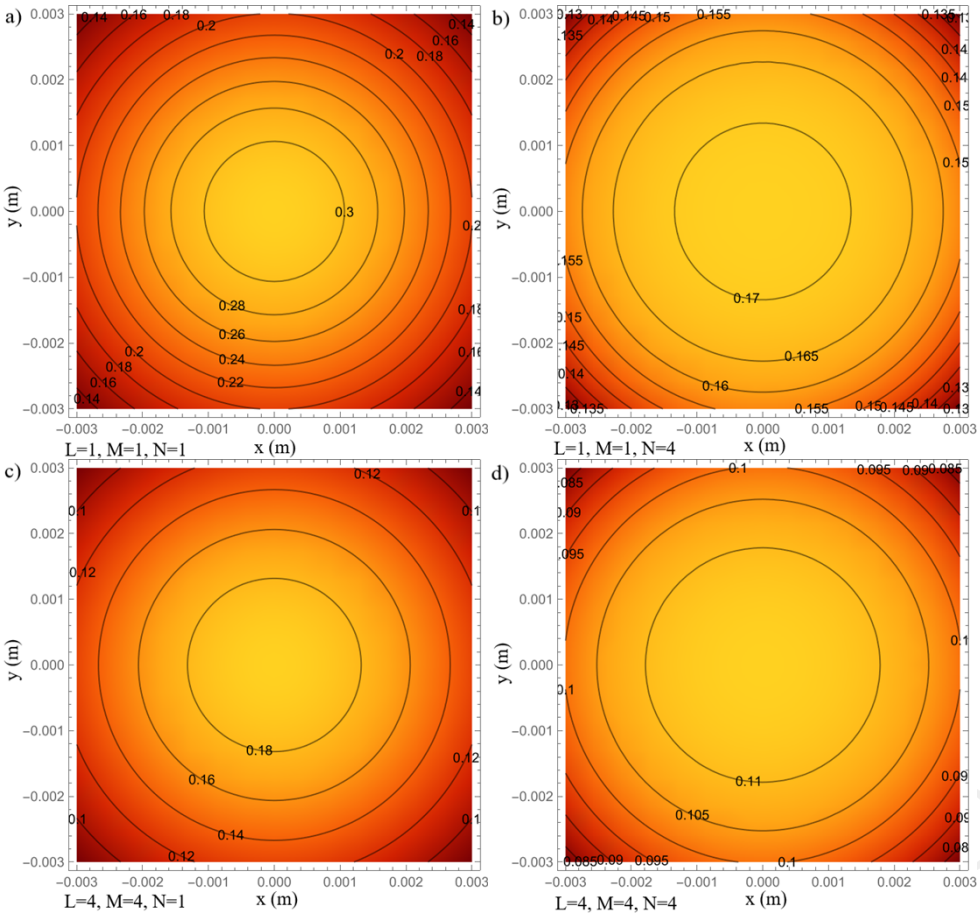


Fig. 2. Density plot of the spectrum of Gaussian Schell-model fields on propagation. The parameters are: $\lambda = 632$ nm, $\sigma = 1$ mm, $\delta_{xx} = 0.15$ mm, $\delta_{yy} = 0.20$ mm, $A_x = 1.5$, $A_y = 1$, $B_{xx} = B_{yy} = 1$, $\varphi = 0$, $z = 5$ m.

Longitudinal and transverse changes in the spectrum of electromagnetic double multi-Gaussian Schell-model beams are analysed in figure 3. Fig. 3(a) and Fig. 3(b) analyses the spectral changes of the multi-Gaussian Schell-model family beams with respect to an increase in the summation index. Flat behavior of the double multi-Gaussian Schell-model beams' spectrum can be seen in Fig. 3(b). Fig. 3(c) and Fig. 3(d) shows the longitudinal and transverse spectral changes of DMGSM beams with respect to a change in the variances of the intensity distribution across the source (σ). Fig. 3(e) and Fig. 3(f) shows the longitudinal and transverse spectral changes of DMGSM beams with respect to a change in the variances of the intensity distribution across the correlations (δ_{ij}). Fig. 3(c) and Fig. 3(e) show a quick decrease in the intensity when the stochastic beams'parameters approach coherent values. Also, comparison of Fig. 3(c) and Fig. 3(e) show a similar response in the spectral change to an increase in σ and δ_{ij}.

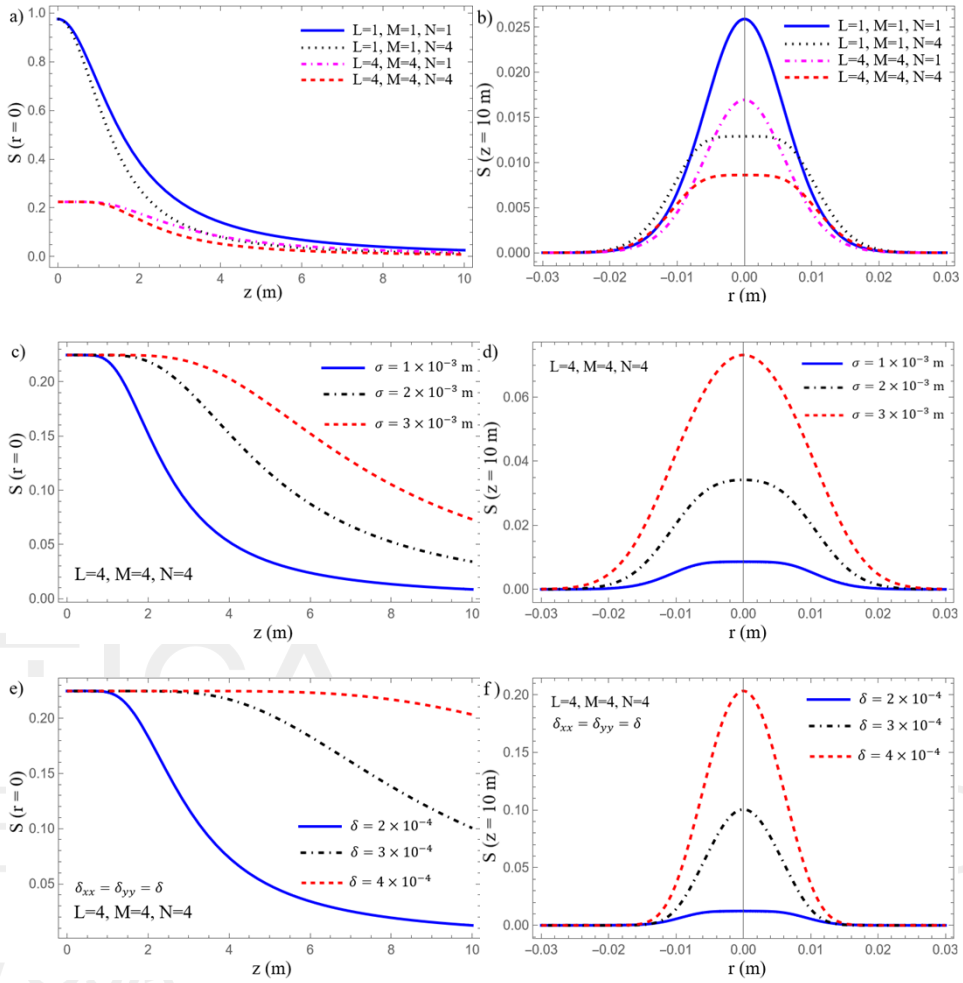


Fig. 3. Normalized longitudinal { (a), (c), (e) } and transverse { (b), (d), (f) } changes in the spectrum of multi-Gaussian Schell-model family beams with the same parameters of Fig. 2 except for σ and δ.

5. Changes in the spectral degree of polarization of double multi-Gaussian Schell-model beams on free-space propagation

The polarization characteristics that result from the electromagnetic field's vector nature are disregarded by scalar fields. Actually, it has been established that the polarization and coherence states of a light field are closely connected and may be discussed as a single phenomenon [4]. Recently, the extension of all varieties of scalar sources to the vector situation has received a lot of attention. The electromagnetic Gaussian Schell-model source is the oldest and most straightforward mathematical model of a stochastic electromagnetic source, and its production, characterization and propagation properties have received much research [25,26].

The degree of polarization and the state of polarization of a random electromagnetic beam at a particular location in space and at a selected frequency make up its polarization characteristics. The formula which expresses the degree of polarization at a point $(\boldsymbol{r},z)$ in its well-known form in terms of the determinant (det) and the trace (Tr) of the cross-spectral density matrix is given by

$$P(\boldsymbol{r},z;\omega) = \sqrt{1 - \frac{4\det \vec{\vec{W}}(\boldsymbol{r},\boldsymbol{r},z;\omega)}{\left[\mathrm{Tr}\, \vec{\vec{W}}(\boldsymbol{r},\boldsymbol{r},z;\omega)\right]^2}}, \tag{32}$$

It is evident from the formula Eq. (28) that the off-diagonal components do not affect the spectral density. But the off-diagonal components are necessary for the spectral degree of polarization analysis. The off-diagonal elements of the cross-specral density matrix obtained from Eq. (26) is given by

$$W_{xy}(\boldsymbol{r},\boldsymbol{r},z;\omega) = \frac{A_x A_y B_{xy}}{C_{0L} C_{0M} C_{0N}} \sum_{l=m=n=1}^{L,M,N} \frac{1}{\Delta_{xy}} (-1)^{l-1} \binom{L}{l} (-1)^{m-1} \binom{M}{m} \frac{(-1)^{n-1}}{n} \binom{N}{n} exp\left[-\frac{4\alpha r^2}{\Delta_{xy}} \right] \quad (33)$$

where Δ_{xy} is given by Eq. (27) and the parameters α, β, and γ_{ij} are given by Eqs. (22-24).

We may calculate the spectral degree of polarization of the field at a point in free space by substitution of Eq. (33) into Eq. (32) together with the substitution of the parameters Eqs. (22-24), and Eq. (27).

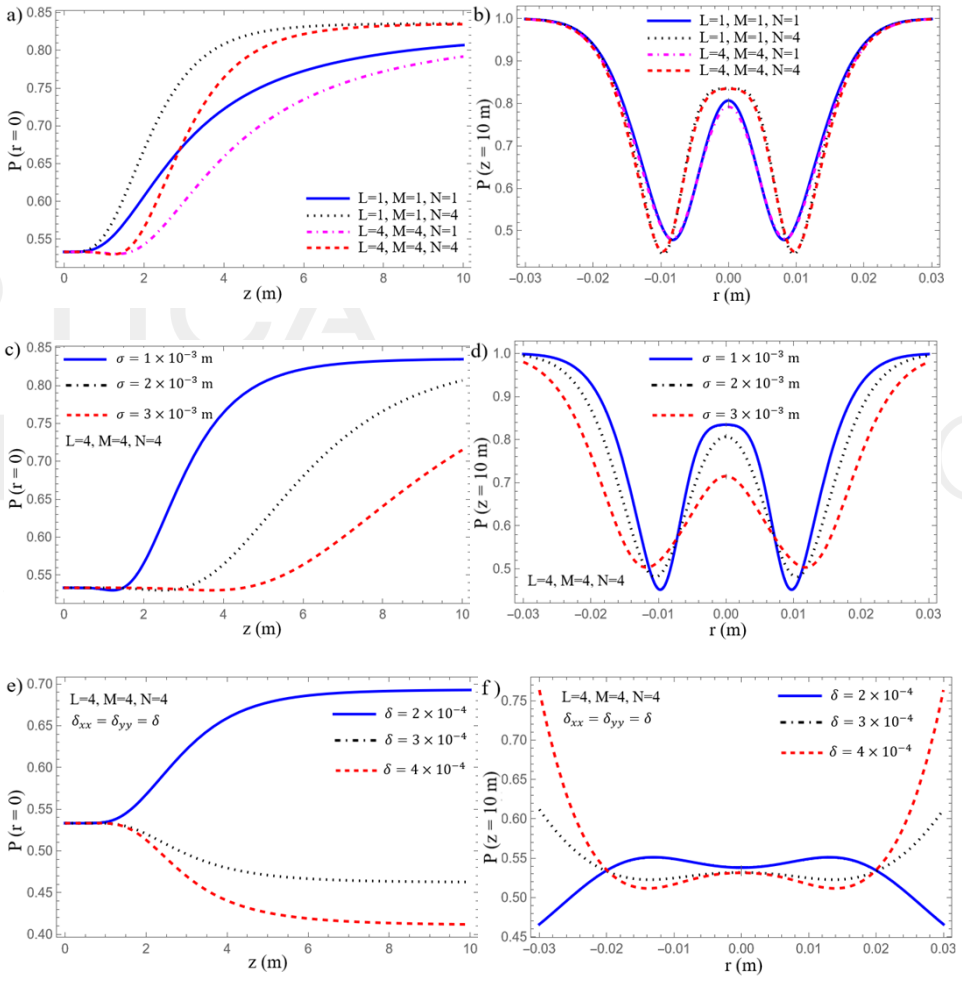


Fig. 4. Longitudinal { (a), (c), (e) } and transverse { (b), (d), (f) } changes in the degree of polarization of multi-Gaussian Schell-model family beams with the same parameters of Fig. 2 except for σ and δ.

In Fig. (4) we illustrate the longitudinal and transverse changes in the spectral degree of polarization of electromagnetic double Multi-Gaussian Schell-model beams propagating through free space. Fig. 4(c) and Fig. 4(e) comparison shows that the same behavior in the

response of DOP to change in sigma and delta is not preserved as it did in the spectrum (Fig. 3(c) vs Fig. 3(e)). An increase in delta results in a decrease in DOP which is seen in Fig. 4(e). Fig. 4(e) and Fig. 4(f) shows that an increase in delta causes a more pronounced change in the DOP both in the longitudinal and in the transverse coordinates. Fig. 3(d) and Fig. 3(f) shows the same trend when sigma and delta change, but DOP does not follow the same trend when sigma and delta change as seen in Fig. 4(d) and Fig. 4(f). Comparison of Fig. 4(d) and Fig. 4(f) shows a change in the distribution when a delta change is considered. One sees in Fig. 4(f) that a delta increase presents a different DOP behavior depending on the parameters.

6. Changes in the spectral degree of coherence of double multi-Gaussian Schell-model beams on free-space propagation

The spectral degree of coherence is the normalized correlation of electric fields which is used for quantifying the coherence between a pair of electric fields. It is typically used to find the statistical character of intensity fluctuations. A quantitative measure of the degree of coherence of a directional (beamlike) field is first introduced, on the basis of the scalar theory, by Zernike [27]. A natural generalization of the spectral degree of coherence for beamlike electromagnetic fields is developed by Wolf in the context of unified theory of coherence and polarization [4].

There are several definitions of degree of coherence including temporal coherence, spatial coherence, and spectral coherence. Temporal coherence refers to the consistency of the phase relationship between light waves over time, while spatial coherence refers to the consistency of the phase relationship between light waves at different points in space. Spectral coherence refers to the consistency of the phase relationship between light waves of different frequencies within a spectrum. The degree of coherence is an important parameter that affects the performance of optical systems and the visibility of interference patterns. One can see [28-31] for detailed discussions of different degree of coherence definitions and the measurements.

The definition of the spectral degree of coherence in space-frequency domain [4], shown explicitly in terms of the spectral density, is given by

$$\eta(\boldsymbol{r_1},\boldsymbol{r_2},z;\omega)=\frac{\mathrm{Tr}\,\vec{W}(\boldsymbol{r_1},\boldsymbol{r_2},z;\omega)}{\sqrt{\mathrm{Tr}\,\vec{W}(\boldsymbol{r_1},\boldsymbol{r_1},z;\omega)}\sqrt{\mathrm{Tr}\,\vec{W}(\boldsymbol{r_2},\boldsymbol{r_2},z;\omega)}}=\frac{W_{xx}(\boldsymbol{r_1},\boldsymbol{r_2},z;\omega)+W_{yy}(\boldsymbol{r_1},\boldsymbol{r_2},z;\omega)}{\sqrt{S(\boldsymbol{r_1},\boldsymbol{r_1},z;\omega)}\sqrt{S(\boldsymbol{r_2},\boldsymbol{r_2},z;\omega)}}. \quad (34)$$

We will specify the two points to be located radially symmetric with respect to the propagation axis; $\boldsymbol{r_2}=-\boldsymbol{r_1}$. As a result we have

$$\boldsymbol{r}=\frac{\boldsymbol{r}_1+\boldsymbol{r_2}}{\boldsymbol{2}}=0, \text{ and } \boldsymbol{r'}=\boldsymbol{r_2}-\boldsymbol{r_1}=2\boldsymbol{r_2}. \quad (35)$$

On substituting from Eq. (35) into Eq. (26), one finds at once the diagonal elements of the cross-specral density matrix that

$$W_{ii}(\boldsymbol{r},\boldsymbol{r'},z;\omega)=\frac{{A_i}^2}{C_{0L}C_{0M}C_{0N}}\sum_{l=m=n=1}^{L,M,N}\frac{1}{\Delta_{ii}}(-1)^{l-1}\binom{L}{l}(-1)^{m-1}\binom{M}{m}\frac{(-1)^{n-1}}{n}\binom{N}{n}exp\Big[-\frac{\gamma_{ii}{\boldsymbol{r_2}}^2}{\Delta_{ii}}\Big], \quad (36)$$

where the parameters α, β, γ_{ii}, and Δ_{ii} are given by Eq. (22), Eq. (23), Eq. (30) and Eq. (31) respectively.

Substituting from Eq. (36) with the parameters into Eq. 34, we obtain the spectral degree of coherence of double multi-Gaussian Schell-model beams on Free-Space Propagation at the chosen pair of points.

Evolution of the modulus of the spectral degree of coherence of electromagnetic DMGSM beams on free space propagation is shown in figure 5. In Fig. 5 the changes in the spectral degree of coherence of double multi-Gaussian Schell-model beams are shown for the case when the pair of field points are located symmetrically with respect to the z-axis (i. e. $\boldsymbol{r_2}=-\boldsymbol{r_1}$). Clearly, the states of coherence of the sources for different order L,N,M have different distributions. The DOC is gradually increasing on propagation with respect to L,M,N, sigma and delta as can be seen in Fig. 5(a), Fig. 5(c) and Fig. 5(e). These differences in source correlated properties lead to different far-field spectral density distributions that was analyzed

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in section 4. As can be inferred from Fig. 5(e) and Fig. 5(f), the DOC does not display a drastical change which is because the DOC is a normalization of the functional behavior of the correlations which is made by introducing the multi-Gaussian form here and it's change with respect to summation indices is shown in Fig. 5(a) and Fig. 5(b).

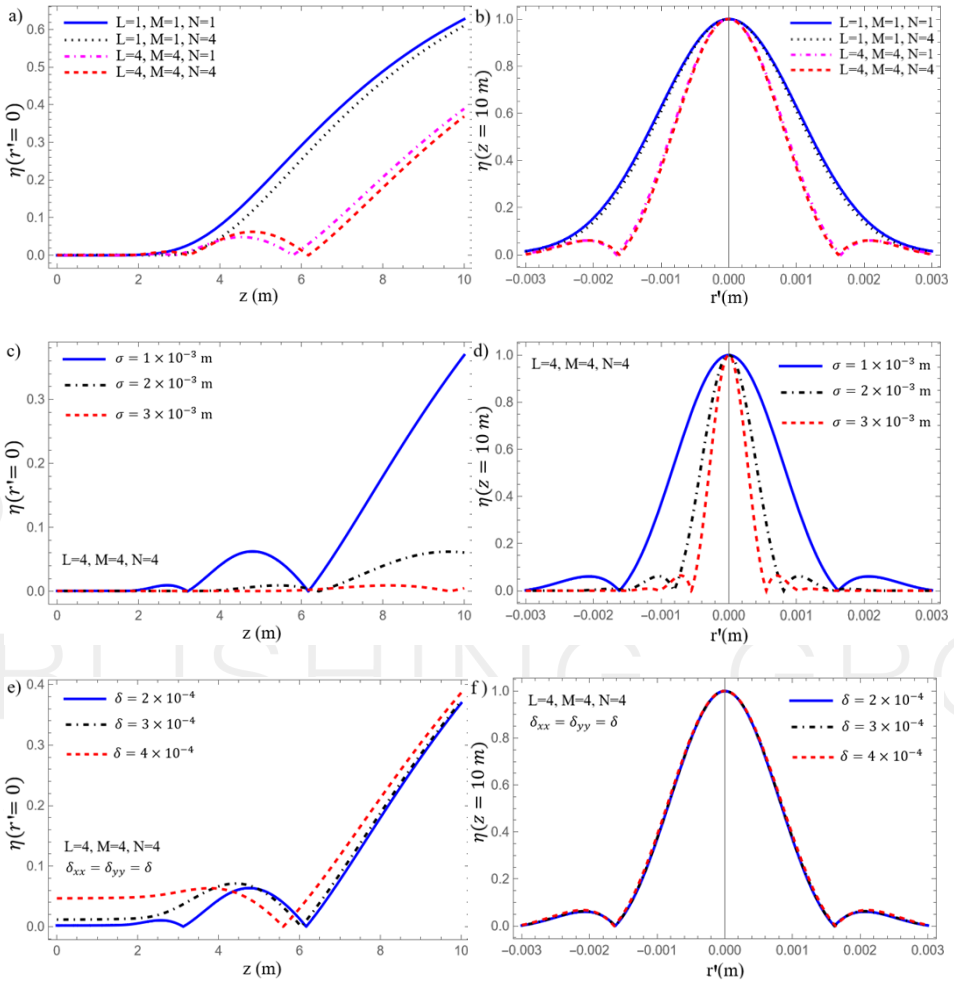


Fig. 5. Longitudinal { (a), (c), (e) }and transverse { (b), (d), (f) } changes in the degree of coherence of multi-Gaussian Schell-model family beams with the same parameters of Fig. 2 except for σ and δ.

7. Conclusion

In summary, general analytic formula for the elements of the cross-spectral density matrix of electromagnetic double multi-Gaussian Schell-model beams propagating in an optical ABCD system have been derived by using the Collins' diffraction integral. We have obtained the expressions of the spectral density, spectral degree of polarization, and the spectral degree of coherence of such beams propagating in free space. We have then examined the changes in the statistical properties of such beams with the help of numerical calculations. The Gaussian Schell-model beam is flattened in the source plane. It is found that the flat behavior of the spectrum is pereserved in the transverse cross-section under sigma and delta variation. Correlation induced increase or decrease in the statistics of Gaussian Schell-model fields can be reversed via modifying the source field with multi Gaussian functions. This brings one more

dimension of freedom in the modeling of Schell-model random light beams. Also, an increase in the correlations parameter (delta) may result in a rise or a drop in the degree of polarization which is a propagation induced polarization change. Genuine cross-spectral density matrices propose a viable, straightforward way for modeling new types of partially coherent fields. The Huygens Fresnel propagation method can be applied to study the propagation of these fields through the turbulent atmosphere or other linear isotropic and homogeneous media.

8. Disclosure Statement

The authors declare no conflicts of interest.

9. Data Availability

Data underlying the results presented in this paper are available in the main text.

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