RUC Semigroups and Their Asymptotic Limit Sets

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Abstract

Strongly continuous one-parameter operator semigroups, or C_0-semigroups, play a central role in solving the abstract Cauchy problem in Banach spaces and analyzing continuous dynamical systems. The classical theory of C_0-semigroups relies on the norm topologies of Banach spaces. In the present work, we focus on spaces where order structures dictate the underlying geometry.

Recent literature [1] has utilized relatively uniform convergence in ordered vector spaces to introduce relatively uniformly continuous (ruc) semigroups. They effectively generalize the topological continuity of classical C_0-semigroups. The use of convergence regulators provides a flexible, order-theoretic approach to analyze semigroup continuity at zero without depending on a standard topological norm.

In this talk, we explore the local properties of trajectories generated by these semigroups and formally introduce the relatively uniform omega (ω_{ru}) and alpha (α_{ru}) limit sets. We establish that under appropriate lattice conditions, such as the property (R), these ru-limit sets exhibit invariance under the semigroup action and satisfy certain closure properties.

Keywords: C_0-semigroups, relatively uniform convergence, asymptotic behavior.

References:

[1] E. Emelyanov, N. Erkurşun-Özcan and S. Gorokhova, Relatively uniformly continuous semigroups on ordered vector spaces. J. Math. Anal. Appl. 560 (2026), no. 2, Paper No. 130556, 13 pp.