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Examining Kindergarten Children's Numerosity Estimation Skills

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Abstract
This study aimed to examine kindergarten children's performance and strategies when answering different types of numerosity estimation questions. To this end, interviews were conducted with 44 children (aged 61–80 months) in an online setting, during which they were asked nine estimation questions of three types. These questions (open-ended, number referent, and visual referent estimation questions) differ in the size of the set to be estimated and in providing reference points for the estimation. The data analysis was based on the content analysis method. The verbal expressions, mimics, and gestures used by the children during the interviews were coded in two stages. In the first stage, the children's estimation performances on different types of questions were evaluated. In the second stage, the strategies used by children for the estimation questions were identified. The estimation performance of children participating in the research was better in number referent estimation questions than in open-ended estimation questions. Moreover, the type of the questions affected the children's strategies. Reportedly, although children tend to use counting strategy in open-ended estimation questions, when a reference point is presented, they move away from these tendencies and turn to more perceptual strategies such as eyeball and benchmark comparison.

Keywords Numerosity estimation · Open-ended estimation · Number referent estimation · Early childhood · Mathematics education · Estimation strategies

Introduction

Estimation has widespread use in the daily lives of children and adults. Each individual, child, or adult may encounter situations that require them to estimate the number of objects in a set, measure of a length, or result of an operation. In addition to discussing the widespread use of estimation in daily life, one may also elucidate its relationship with other mathematical skills, such as arithmetic skills or general mathematics achievement (Dowker, 2003). Prior research suggests that students who are adept at estimation demonstrate better arithmetic skills in terms of counting, number sense, mental computation, strategy flexibility, and conceptual understanding (Booth & Siegler, 2006; Crites, 1992; Luwel & Verschaffel, 2008; Sowder, 1992). Several studies (Baroody & Gatzke, 1991; Crites, 1992; Hogan & Brezinski, 2003; Lang, 2001; Xu et al., 2013) have suggested estimation as a distinct skill, whereas others (Jordan et al., 2006; Lago & DiPerna, 2010) have considered it a component of number sense. The estimation process is a comprehensive process that varies according to the variety of tasks, required knowledge, and types of estimation.

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Types of Estimation

Sowder (1992) identified three estimation categories based on the intended purpose. Computational estimation requires estimating the result of an operation by performing mental calculations using approximate values of the given numbers. Measurement estimations, which are conducted when making an estimation of a measurement, can relate to measurements including length, perimeter, area, and volume. Numerosity estimation refers to situations wherein the numerical value of a certain quantity is to be estimated without using an operation or measurement unit, for example, estimating the number of marbles in a jar. Siegler and Booth (2005) added number line estimation to the aforementioned three types of estimation, wherein the importance of the number line in the processes of making sense of numerical quantities is underlined. Depending on the intended purposes of the estimation, computational estimations require knowledge of calculation; measurement estimations require knowledge of measurement units; and numerosity estimations and number line estimations require knowledge of numbers (Siegler & Booth, 2005).

Numerosity Estimation

The lack of knowledge regarding the estimation skill levels of school-age children renders it difficult to structure the teaching of estimation skills (Baroody & Gatzke, 1991). The first problem concerns the number of items required in the set to be estimated. Van de Walle and Thomson (1985) posited that the numerosity estimation experience should begin with 4–12 objects, while Leutzinger, et al. (1986) stated that the estimation range should be between 10 and 30. Douglass (1925) noted that from the age of four-and-a-half years, children can recognize up to five objects without counting (subitizing) but are unable to do so for 6–12 objects. This finding indicates that estimation exercises with this age group should involve more than five objects.

In the preschool-kindergarten period, the concept of estimation can be practiced in the following two ways: estimating set sizes using reference points, or approximating set sizes without reference points. In approximate estimation tasks without reference points, students are asked to make open-ended estimations, whereas in estimation tasks with reference points, they are asked to estimate whether the number of a group of objects is smaller or larger than a given number, or whether it is less than, in between, or more than two given reference points (e.g., less than 10, between 10 and 20, or more than 20) (Baroody & Gatzke, 1991). Rubenstein (1985) claimed that estimation without reference points is more difficult than estimation with reference points.

Studies on estimation have increased especially in recent years. In these studies, examining the strategies and performances used in different types of estimation (Baran Kaya & Çelik Demirci, 2022) and the handling of estimation in the curricula of different countries stand out (Andrews et al., 2022; Bødtker Sunde et al., 2022). Some studies have attempted to determine the different strategies that students use in their numerosity estimation processes (Crites, 1992; Lang, 2001; Siegel et al., 1982). However, arguably, studies focusing on numerosity estimation are limited. Brumm and Rathgeb-Schnierer (2022) stated the research gaps related with the relationship between estimation strategies and accuracy in various types of tasks, task-specific influences on the choice of strategy, and the accuracy of estimated values. Accordingly, examining the strategies used by students while estimating, combined with the tasks will be useful in ascertaining students' estimation process.

Present Study

This study aims to examine kindergarten children's estimation strategies in various types of tasks and task-specific influences on the choice of strategy as well as the performance (estimated values' accuracy) of children. The following research questions guided the research process:

1. How do kindergarten children differ in their performance with respect to different types of numerosity estimation questions?
2. What strategies do kindergarten children use when answering different types of numerosity estimation questions?

Methods

Participants and Setting

This study included 44 children (22 females; 22 males) attending kindergarten in six public schools run by the Ministry of National Education, catering to families of medium socioeconomic status in the Urla district of the province of İzmir, where one of the researchers works. The children participating in the study were selected using a criterion sampling approach, a purposive sampling method. Children younger than 60 months were not included in the study, considering the difficulty in obtaining data from children who have not received any formal education prior to kindergarten. In Turkey, kindergarten classes cater to 57–68 months-old children as of the end of September. The criteria used in the selection of the children to be interviewed included aged > 60 months, attending kindergarten in the 2020–2021 academic year, and volunteering to participate in an online

interview. The participants' age varied between 61 and 80 months, and the average age was 72.5 months. No learning outcome exists for the estimation of quantity in the early childhood curriculum implemented in Turkey; hence, children did not have any formal prior experience with such estimation. Owing to the pandemic, children have been unable to receive regular face-to-face in-school education for the last two years. Schools offered mandatory distance education in the spring semester of the 2019–2020 academic year, and a hybrid of face-to-face and distance education in the 2020–2021 academic year.

Data Collection

Relevant permissions were obtained from the University and National Education Ethics Commissions prior to commencing data collection. Before conducting the interviews, the parents of all the children attending kindergarten in the selected schools were contacted by the respective school administrators and teachers; detailed study information was provided via e-mail.

The interviews were conducted online owing to the ongoing pandemic in the spring semester of the 2020–2021 academic year using the Zoom application. The children's online interviews were conducted without any difficulties because they were accustomed to attending online activities with their teachers as part of their distance education. Individual interviews with children, accompanied by their parents, lasted on average 20 min, and were conducted by one of the researchers who had teaching experience.

Prior to the interviews, preliminary meetings were held with the children to allow them to develop a rapport with the researcher, ensuring their comfort during the interview. During these preliminary meetings, conducted through online video calls, the researcher introduced herself to the child, had a brief chat, talked about the character "Zıpzıp" mentioned in the interview questions, and invited the child to participate in an online interview wherein they would help this character in various games. Prior to the interviews, no training in estimation skills was provided to the children or their parents.

During the interviews, care was taken to maintain a friendly rapport with the children to ensure that they felt comfortable and did not become bored. To ensure that the children were comfortable to express themselves freely, the topics of these chats included their favorite toys, colors, foods, animals, clothes, and hairstyles, depending on their interests and preferences. During the interviews, the researcher presented the interview questions using the PowerPoint program in the form of a game, with a different visual on each slide (Appendix 1). In the interviews, the children were introduced to the Zıpzıp character, who is curious and loves to play games. The participants were told that Zıpzıp could only be helped by children, and that they could help him by responding to questions and explaining their thinking.

To ensure reliability, all the interviews were conducted by the same researcher, and care was taken to avoid directing children toward the correct answers. Moreover, the accompanying parents were asked prior to the interviews to avoid helping their children, or to state anything during the interviews that could serve as clues or reveal the correct answers.

Data Collection Instrument

The discussions and questions used in related literature and the pilot study's results were considered to develop and finalize the interview questions (Baroody & Gatzke, 1991; Crites, 1992; Lang, 2001; Leutzinger et al., 1986). Although the literature has documented that the size of the set to be estimated should be 30 for the preschool period, the size can be determined depending on the children's familiarity with these numbers (Leutzinger, et al., 1986). Additionally, it is emphasized that estimation activities should be commenced with a set having number of elements larger than five, because young children can subitize a set of objects smaller than five. Therefore, the estimation questions in this study comprise three types that require the estimation of different number of objects set and nine questions. Developing different types of questions is important to investigate children's estimation performances and the strategies that they use in relation to the questions' characteristics. The questions differ in set sizes and in providing reference points for estimation. The questions in the first and second parts asked the children to make numerosity estimations, for which they were presented three images containing 12, 20, and 90 apples for five seconds each, and asked the following questions: "About how many apples do you think there are on this tree? What was your thinking?" The children were asked to estimate the number of objects in the images and explain their thinking to Zıpzıp. For the second type of question, reference points were offered for example, "Is the number of objects closer to 5, 10 or 20? " The children were asked to estimate which of the three numbers was closest to the actual number of objects. In the third type of question, the children were asked to estimate the number of cubes in structures of unifix cubes. These questions used cubes of three different colors (yellow, blue, and red). The children were presented a picture containing five unifix yellow cubes on the left-hand side. The researcher told the interviewee that there were five yellow cubes in the visual and asked them to estimate the number of unifix cubes in the other colors on the visual's right-hand side. For example, in these types of questions, the children were asked, "There are five yellow cubes here, how many red cubes do you think there are? How did you think? " The yellow cubes were included to observe whether they were used as reference points by

the children. In short, type-1 and type-3 questions asked the children to estimate the number of objects in the visual, and the children were expected to provide an estimated number on their own, while in the type-2 questions, the children were asked to select a number from among three reference points.

Pilot Study

Prior to data collection, a pilot study involving five children aged 60–72 months was conducted. The pilot study indicated that the sizes, distributions, and arrangement of the objects affected the strategies used by the children, hence, a decision was made to use objects of identical size in all questions, and to present them within a frame.

Furthermore, the pilot study revealed that keeping the visual on the screen for a long time encouraged the children to count, and a decision was made based on this observation to keep each visual on the screen for only five seconds, to be replaced by a white screen while waiting for the child to make their estimation.

Data Analysis

Data from the study comprised the records of the interviews conducted with the children (audio and video) and researchers' observation notes. The verbal expressions, mimics, and gestures (such as using fingers for counting) used by the children during the interviews when they responded to estimation questions were recorded in a Word document. The data obtained from the children interviewed were transferred to separate tables by researchers. In addition to the scores, the tables comprised the critical statements, behaviors, and estimations of the children, as well as the strategies that they used.

Coding was conducted in the following two stages: In the first stage, the children's estimation performances were scored and evaluated using a rubric created for the purpose. To evaluate the estimation questions, an interval was created based on prior studies. The estimation questions were evaluated based on the 25% deviation rule used in other studies (Baroody & Gatzke, 1991; Crites, 1992). Accordingly, in type-1 questions, responses that deviated from the actual value by a maximum of 25% (actual value $\mp$ % 25) were assigned a score of 2, those that deviated from the actual value by 50% were assigned a score of 1, and those that deviated from the actual value by more than 50% were assigned a score of 0. For example, for the type-1 question related to a visual containing 12 apples on the tree, responses between 9 and 15 were assigned a score of 2; responses between 6 and 8, and between 16 and 18 were assigned a score of 1; and responses between 1 and 5, and 19 or above were assigned a score of 0. Appendix-1 presents the rubric for each question.

A content analysis method was used to record children's strategies in different types of estimation questions. In the second stage, the interview transcripts were examined in-depth by all the researchers in order to analyze the strategies used by the children in the estimation questions. Strategies for estimation questions determined by Siegel et al. (1982) formed a basis for this study's data analysis. First, tables comprising the children's answers to each interview question were created separately by all the researchers. In these tables, the students' verbal expressions formed the initial codes. Thereafter, the researchers collaborated and discussed the similarities and differences in the expressions of the children and gathered the codes that they determined under various themes. In rare coding differences, the related student's video recording was watched and discussed by all the researchers together, and a consensus was reached. Following the literature, strategies used by children for answering estimation questions were grouped under the headings of eyeball, guess, benchmark comparison, range, measurement, subitizing, counting and unknown. The critical statements and behaviors related to the strategies used by the children in the estimation questions during the interviews are presented in Table 1 with direct quotations. For example, children using the counting strategy counted the objects as far as they could (come) during the five seconds that the image was presented and continued making counting movements with their eyes, head, or fingers when the object to be guessed was removed from the screen. The first question that the researcher asked was "How did you count?" A child tried to take a guess by thinking about the positions of the pictures in his mind and stated, "I count as if there is a tree with my hand when there is no picture". Alternatively, children using the eyeball strategy provided more perceptual explanations using expressions such as "more" and "less."

Findings

Findings Related to the First Research Question

To answer the first research question, a rubric was used to evaluate the overall performance of the children who participated, by question type. In this rubric, the highest possible score that could be obtained from the test was 18. Table 2 presents descriptive statistics concerning the children's estimation performances by question type.

Table 2 reveals that the mean score received by the children across the entire test was 11.6 out of 18. Children received a mean score of 3.1 out of 6 on the type-1 questions, a mean score of 4.9 out of 6 on the type-2 questions, and a mean score of 3.7 out of 6 on the type-3 questions. The children's performance on the type-2 questions were notably higher than on the other two question types. An analysis of

Table 1 Strategies used in estimation questions

Strategies	Strategy description	Sample student responses
1. Guess	Children simply think of an answer to the question and share it. They cannot make an explanation regarding the quantity	"I decided by looking." "I know it."
2. Eyeball	Children make **a perception-based description** to explain their estimate. They use phrases such as "a lot" and "few" instead of comparing with an exact number	"Because there were quite a lot in there." (Question 2) "That must really be 100, because there were a lot." (Question 3)
3. Benchmark comparison	Children estimate by **making a comparison with another quantity**	"Because there must be **more**. There were more red cubes. Yellow cubes were fewer. Because when you match it with the yellow cube, the red cube is bigger" (Question 10)
4. Range	Children define **a limited range** in their estimations. The reference strategy involved a comparison with a single number; however, in this strategy, a lower threshold and an upper threshold are defined for quantities	"It would be too few if it were 5. 10 was in the middle. That's why" (Question 4)
5. Measurement	In their estimates, children focus on the area covered by the objects rather than the objects themselves	"It appears to be 100 but it doesn't seem to be 100. Because it covers a lot of the tree" (Question 3)
6. Counting	Children make counting movements with their eyes, head, or hands, or imagine the visual in their minds, and count even when they cannot see the visual	"You would know if you counted, I counted in the beginning." Child: "I counted 10 Researcher: How did you count? There is no picture Child: I counted using my hand when there is no picture, like this. I saw it briefly, but I know, I can see" (Question 1)
7. Subitizing	Children perceive numbers by dividing them into pieces (conceptual subitizing)	(Gave the answer 12 very quickly) "Because there were 6 in one place, and 6 in the other place. I brought them together, and it was 12" (Question 1)
8. Unknown	This refers to situations wherein children do not offer explanations regarding their thinking; in such a case nothing can be inferred from their behaviors	Children stay silent, shrug or start talking about another topic

Table 2 Descriptive statistical results of children's estimation score according to question types

QUESTION TYPES	N	MIN	MAX	$\overline{X}$	SD
TYPE 1	44	0	6	3.1	1.7
TYPE 2	44	2	6	4.9	1.3
TYPE 3	44	0	6	3.7	1.9
TOTAL	44	2	18	11.6	3.5

the total scores from the test revealed a moderate estimation performance from the children.

Table 3 presents the distribution of the children's estimation performances (frequencies and percentage values) by individual questions. As Table 3 indicates, among the open-ended estimation type-1 questions, the children performed optimally in the first question, wherein there were 12 apples on the tree. More than half of the students (59.1%) provided an answer in the range of 9–15 as their estimations. Among the type-1 questions, the lowest performance was observed in the third question, wherein there were 90 apples. One notable observation regarding the responses to this question was that upon seeing it, the students opened their eyes or used verbal expressions such as "That is a lot!" or "This is difficult,", which can be considered indicators of quantity-related perception. Overall, 70% of the children made estimates ranging from 1 to 44 in response to this question, and only 20% provided answers in the 65–110 range. One of the children, in attempting to provide an estimate, thought about the largest number they knew and stated, "This is somewhat difficult. I forgot the name of the largest number. I can't think of anything other than 20" (Child 2).

Among the most notable findings of this study was that children performed better in type-2 questions than in type 1 and 3 questions. Although the number of apples remained the same, the children achieved better scores in question six (type-2) than in question three (type-1) with 86% of the children selecting the "more than 20" option to a question wherein the image had 90 apples on the tree. Among the questions of this type, their performance related to the question wherein there were 12 apples was somewhat lower. Among the number referent estimation questions, several children selected the "close to 10" option for the fourth question, in which there were 12 apples, one making an explanation based on perceptions of quantity saying, "Because these is a little bit more, and a little bit less" (Child 1).

However, in the type-3 questions, the students recorded medium performance when compared with the other types of questions. These questions differed from other questions, both visually, and in the fact that they offered another quantity serving as a reference point. In type-2 questions, the children were expected to select an approximate amount from the reference points offered, whereas in type-3 questions, they were expected to define a new quantity by comparing it with another quantity. Among the questions of this type, more than half of the children recorded satisfactory performance, particularly on questions 7 and 9. One of the children who made a correct estimation in question 8, which had a lower success rate, estimated the number of red cubes as six, and explained their answer as follows: "Because yours always seem to have five (referring to the yellow cubes), mine has only one more than yours. This is how I knew" (Child 5). This student made an estimation using the number of yellow cubes as a reference point.

Findings Related to the Second Research Question

To answer the second research question, the strategies used by children to answer the estimation questions were examined by question type. Table 4 presents the frequency and percentage values of the strategies used by children for all three question types. As Table 4 indicates, the most frequently used strategy for all three question types was guess. In this strategy, children simply think of an answer but are unable to provide a perception-based explanation. For example, some of the explanations used in the interviews were "I thought so" and "Because there must be that many."

The most frequently used strategy in response to type-1 questions, which required making an open-ended estimation, was guess. After guess, the second most popular strategy was counting. Approximately 26% of the children made counting movements with their eyes, lips, or fingers when responding to the estimation questions, for which they moved closer to the screen. Certain students continued counting with their fingers and pointed to the screen even after the visual was removed at the end of five seconds, apparently keeping the visual in their minds. One of the students attempted to count all the apples on the tree one by one, making the following comments: "1, 2, 3, 4, 5… Where did it go? I am not done counting" (Child 20). The tendency to count was stronger in questions one (12 apples) and two (20 apples), and somewhat weaker in the third question, which featured a larger number of apples.

For the type-2 questions, the second most popular strategy after guess was the eyeball strategy with 30% of the children providing explanations based on their perception of quantities for their estimations. Notably, children who used the eyeball strategy made statements and explanations regarding quantities during the interviews, such as, "That must really be 100, because there were a lot." As Table 4 indicates, the eyeball strategy was the second most popular approach after guess in all three type-2 questions. Children who used this strategy attempted to describe the given quantity using phrases such as "a lot" and "a few", instead of making comparisons with exact numbers. For example, in response to question five, one of the children selected the option "closer to 20", and explained their

Table 3 Distribution of the children's estimation performances (frequencies and percentage values) by individual questions

	Type 1 Open-ended estimation			Type 2 Number referent estimation			Type 3 Visual referent estimation		
	Set size								
	12	*20*	*90*	*12*	*20*	*90*	*4*	*6*	*10*
SCORE	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9
	f (%)	f (%)	f (%)	f (%)	f (%)	f (%)	f (%)	f (%)	f (%)
0	11 (25)	11 (25)	31 (70.5)	11 (25)	2 (4.5)	2 (4.5)	9 (20.5)	19 (43.2)	16 (36.4)
1	7 (15.9)	18 (40.9)	2 (4.5)	10 (22.7)	8 (18.2)	4 (9.1)	7 (15.9)	5 (11.4)	5 (11.4)
2	26 (59.1)	15 (34.1)	11 (20.5)	23 (52.3)	34 (77.3)	38 (86.4)	28 (3.6)	20 (43.5)	23 (52.3)

Table 4 Distribution of the children's strategies (frequencies and percentage values) by question types

Question types Strategies	Type 1 f (%)	Type 2 f (%)	Type 3 f (%)
Guess	45 (34.9)	41 (31.8)	34 (26.4)
Eyeball	27 (20.9)	38 (29.5)	17 (13.2)
Benchmark comparison	5 (3.9)	6 (4.7)	33 (25.6)
Range	1 (0.8)	4 (3.1)	3 (2.3)
Measurement	2 (1.6)	0 (0)	5 (3.9)
Counting	33 (25.6)	11(8.5)	32 (24.8)
Subitizing	5 (3.9)	0 (0)	1 (0.8)
Unknown	10 (7.8)	28 (21.7)	4 (3.1)

reasoning by saying "There were a lot of fruit, and the number 20 is also a lot" (Child 7).

However, in the type-3 questions, benchmark comparison was among the most frequently used strategies, wherein children utilized the yellow cubes to estimate the number of other cubes. For example, estimating the number of blue cubes in question seven, one of the children said, "Because there were five over there (referring to the yellow cubes), while over here, it was one less" (Child 5). The children who did not utilize the yellow cubes, tended to estimate by trying to count the blue cubes one-by-one.

Discussion

Similar to other studies (Baran Kaya & Çelik Demirci, 2022; Baroody & Gatzke, 1991; Brumm & Rathgeb-Schnierer, 2022; Crites, 1992; Rubenstein, 1985), our study found that the characteristics of question types affected children's estimation processes. This study's results revealed that the type of questions for which the children participating in the study performed optimally were the number referent estimation questions. Notably, although both question types used the same set sizes, the children performed better with type-2 than with type-1 questions. Differences were noted between these two question types in terms of the strategies used. The better performance when faced with number referent estimation questions can be partially attributed to the children moving away from the tendency to use counting in these question types and beginning to use eyeball strategy. Therefore, presenting reference points in the form of "Is it closer to 5, 10 or 20?" may have led children to use different strategies and, thus, perform better.

Among the type-2 questions, the estimation performance was the highest in the question with the set size of 90, as most children selected the option "more than 20" in response to this question. For children of this age, numbers larger than 20 are perceived as "many." According to Baroody and Wilkins (1999), "the numbers 30 and 100 are the same size to a child: they are many." In fact, when asked to name a number larger than 20 during the interviews, the children provided inconsistent answers. For example, one of the children said 30, whereas another said 40,000. The satisfactory performance in response to this specific question may be attributed to this perception of quantities. Among the questions of this type, the worst performance was observed with question four with the set size of 12. One reason for the low performance on this question may be that the number of objects to be estimated (12) was close to the reference points provided (5, 10, and 20). By contrast, the number of objects to be estimated (90) in question six was distant from the given reference points (10, 20, and more than 20), which may have improved the performance in this question and affected the strategy selection. The eyeball strategy was the most popular strategy in this question, wherein the children provided perception-based descriptions of the number of apples on the tree using expressions such as, "Wow, this is a lot of apples."

Among all the estimation questions, the lowest performance was recorded for open-ended estimation questions of type-1. An interesting finding of this study was that unlike with type-2 questions, the larger set sizes in this question type were associated with lower performance. When they were asked to estimate the number of apples on the tree (90) in question three, the children faced difficulty in naming a number that represents this quantity. Most children answered 20 in response to this question, which can be attributed to the knowledge of numbers that children in this age group are expected to have. For children of this age, the curriculum

sets the limit of their knowledge of numbers at 20 [Ministry of National Education (MoNE), 2013]. Therefore, their low performance in open-ended estimation questions cannot be attributed directly to them being unable to perceive quantities represented by large numbers. Therefore, providing context for quantities represented by large numbers (e.g., the number of stars in the sky being in millions; one cannot eat 100 olives for breakfast) would help children associate these numbers with their own lives and make better sense of them. The goal here is not to teach the number 100 in a symbolic manner but to make children understand when a quantity corresponds to the number 100. In fact, children who had had meaningful experiences with large numbers provided examples from cartoon films, books, and family games.

The type-3 questions differed from the type-1 and -2 questions in that the quantity to be estimated and the quantity to be used as a reference point were presented together in the same visual. In questions of this type, unlike type-2 questions, children were offered a quantity other than reference points and were expected to make a comparison. One interesting finding was that the children used different estimation strategies. The use of a benchmark comparison strategy was more common in this question type than in other question types with children attempting to estimate the number of unifix cubes by benchmarking them to the yellow cubes provided in the visual.

An important finding of the study was that the different strategies defined in literature were also observed among kindergarten children, including guess, eyeball, benchmark comparison, and range strategies, among others. Similar strategies were observed in studies with elementary and middle-school students, conducted by Crites (1992) and Siegel et al. (1982). Unlike other studies, this study found that the measurement strategy was also used, wherein children do not focus on the number of objects to be estimated, but instead focus on their arrangement (the area covered by apples on the tree) or the height of the object to be used as a reference point (the yellow cubes' height). Future studies may provide deeper insight into how differences in the arrangement of the set of objects in the same area affect this strategy's utilization. Markovits and Hershkowitz (1993) found that the sequence of the objects to be predicted affects the strategies used by the children.

The most popular approach in all question types was the guess strategy, which was explained by children using expressions such as "It seemed that way" or "It appeared to me that way." In this strategy, children did not make comments regarding the quantities. Clearly defining the estimation processes employed herein is difficult because children could not provide an adequate explanation while estimating. Some children's inability to provide perception-based explanations for estimations made using the guess strategy may be attributed to their lack of experience with estimation. According to Micklo (1999), the earlier children are exposed to estimation experiences, the more they would value and effectively utilized the techniques.

Limitations

A limitation of this study is related to the results' generalizability. The data obtained from 44 children interviewed cannot be generalized to all kindergarten children. However, asking three different types of estimation questions provided an opportunity to identify the differences and similarities in the children's answers to these questions. Therefore, this study's results provide important clues regarding the strategies that children use when answering numerosity estimation questions. Another limitation of the study pertains to the collection of data through online interviews owing to the pandemic. In this study, the object to be predicted was not presented physically to the children; instead, the images were presented online on a PowerPoint screen. Future studies can examine whether physically presenting the object to be predicted affects the children's estimation performance.

Implications

The grade level at which estimation teaching should begin, the upper and lower thresholds for the size of the sets to be used for estimation, and the type of questions to ask are matters of scholarly debate (Baroody & Gatzke, 1991; Baroody et al., 2005; Crites, 1992). This study's findings provide insights into kindergarten children's numerosity estimation skills, which can be important in designing developmentally appropriate plans for teaching estimation skills in early childhood. The finding that children use different strategies and make more meaningful estimations when faced with estimation questions with reference points, which was not the case in open-ended estimation questions, supports Baroody and Gatzke's (1991) argument that estimation activities with reference points would be more appropriate for the teaching of estimation at the preschool-kindergarten level, particularly when the set sizes are larger than 25.

As in the mathematics curriculum of Denmark, Norway, and Sweden (Bødtker Sunde et al., 2022), the Turkish Early Childhood Education Curriculum contains no learning objectives or activities involving numerosity estimation, except measurement estimation (MoNE, 2013). Therefore, to support the development of children's estimation skills, learning objectives and descriptions involving numerosity estimations should be added to the Turkish Early Childhood Education Curriculum. Classroom activities supporting these learning objectives may include providing reference points and asking children to estimate set sizes, or providing them the number of marbles in a jar and asking them to estimate the number of marbles in another jar using the first jar as a reference. Simply asking children to guess the number of marbles in a random jar would not allow them to internalize the estimations' meaning (Lang, 2001; Micklo, 1999). Games played in digital settings can also be used as an educational tool by families and teachers to support

children's estimation skills (Can, 2020). Papadakis et al. (2018) concluded in their study that the use of tablets with developmentally appropriate software in children's daily routines is effective in children's understanding of numbers.

Appendix-1

See Table 5

Table 5 Interview questions and rubric

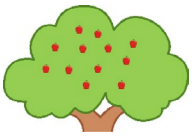
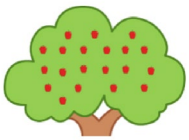
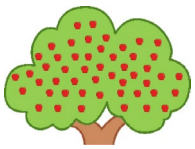
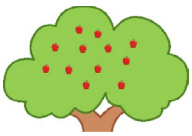
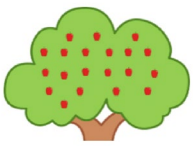
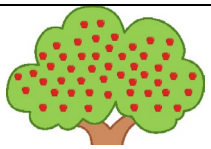
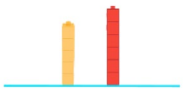
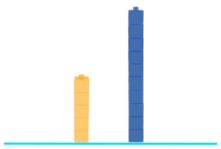
Question type	Set size	Question	Image of the questions	Rubric
Type 1: Open-ended estimation	12	Question 1: About how many apples do you think there are on this tree? What was your thinking?"		Responses "between 1 and 5 and 19 or above" were assigned a score of 0. Responses "between 6 and 8, and between 16 and 18" were assigned a score of 1 Responses of "between 9 and 15" were assigned a score of 2
	20	Question 2: About how many apples do you think there are on this tree? What was your thinking?		Responses "between 1 and 9 and 31 or above" were assigned a score of 0. Responses "between 10 and 14, and between 26 and 30" were assigned a score of 1 Responses of "between 15 and 25" were assigned a score of 2
	90	Question 3: About how many apples do you think there are on this tree? What was your thinking?		Responses "between 1 and 44 and 136 or above" were assigned a score of 0. Responses "between 45 and 64, and between 111 and 135" were assigned a score of 1 Responses of between "65 and 110" were assigned a score of 2
Type 2: Number referent estimation	12	Question 4: Do you think the number of apples on this tree is closer to 5, 10 or 20?		Those who answer "closer to 5" 0 point Those who answer "closer to 10" 2 point Those who answer "closer to 20" 1 point
	20	Question 5: Do you think the number of apples on this tree is closer to 5, 10 or 20?		Those who answer "closer to 5" 0 point Those who answer "closer to 10" 2 point Those who answer "closer to 20" 1 point
	90	Question 6: Do you think the number of apples on this tree is close to 10, close to 20, or more than 20?		Those who answer "closer to 5" 0 point Those who answer "closer to 10" 2 point Those who answer "more than 20" 1 point

Table 5 (continued)

Type 3: Visual referent estimation	4	Question 7: There are 5 yellow cubes here. How many blue cubes do you think there are?		<u>For blue cubes:</u> Responses "5 and above" were assigned a score of 0. Responses "1 and 2" were assigned a score of 0. Responses "3" were assigned a score of 1. Responses "4" were assigned a score of 2.
	6	Question 8: There are 5 yellow cubes here. How many red cubes do you think there are?		<u>For red cubes:</u> Responses between "0 and 5" were assigned a score of 0. Responses "6 and 7" were assigned a score of 2. Responses "8 and 9" were assigned a score of 1. Responses "10 and more" were assigned a score of 0.
	10	Question 9: If the number of yellow cubes is 5, what is the number of blue cubes?		<u>For blue cubes:</u> Responses between "0 and 6" were assigned a score of 0. Responses "7 and 8" were assigned a score of 1. Responses "9, 10 and 11" were assigned a score of 2. Responses "12 and 13" were assigned a score of 1. Responses "14 and more" were assigned a score of 0.

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